Scaling relations for telescopes, spectrographs, and reimaging instruments

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1 Introduction

To make modern astronomical observations, one needs a telescope and a detector, but generally also an instrument to modify the light delivered from the telescope. The telescope brings light from near-infinity to a focus on its focal surface. For large optical/IR telescopes, the focal length at the secondary focus (Cassegrain/Gregorian/Nasmyth) is quite large, and so is the diameter of the focal surface. While it is possible to put a detector directly at the secondary focus, the sizes of modern electronic detectors are much smaller than the focal surfaces of large telescopes, and so reimaging instrumentation is used to demagnify the image. (The major exceptions are imaging cameras placed at the prime focus of the telescope.)

In order to do wide field imaging or nearly any kind of spectroscopy, we typically place a reimaging instrument behind the focal surface, and slits at the focal surface, if needed for spectroscopy. The instrument usually consists of a collimating lens, a dispersing element in the collimated beam if desired, and a camera that reimages the collimated beams onto a detector.

This document attempts to explain some basics of these reimaging systems and to derive some scaling laws. This is not an optics text. There are many existing texts, but few specifically treat astronomical applications and many optics texts drop the reader directly into complexities such as the mathematics of aberrations. For further background and more detail, see for example *Astronomical Optics* by Daniel Schroeder; classic articles on issues of spectrograph design include those by I.S. Bowen (1964, volume I of *Stars and Stellar Systems*) and R.G. Bingham (1979, QJRAS, 20, 395).

The reason for writing this is, in part, that observers have become more disconnected from the instrumentation as it becomes more complex. On my first observing run, I used an instrument that my advisor built, which the two of us could pick up, and take off the side panels to look at the optical path and adjust the internal focus. Today, there are still some hands-on opportunities, but nobody is going to let a green grad student put his or her hands inside a 8-meter class instrument. So the chance to see how things work is increasingly restricted to instrument builders and optical designers. For them, this is basic lore that "everybody knows," but it rarely gets taught in a basic optics class.

2 Reimaging systems

2.1 Telescopes and plate scale

Some definitions:

f = focal length, mm
N = f/number: ratio of focal length to aperture or beam diameter
D = physical diameters, mm
s = scale at a focal plane, arcsec/mm
θ_{field} = angular field of view

To simplify the optical analysis, I will consider optical elements that are focused at infinity. We will deal with mirrors and lenses that take incident collimated beams - parallel ray bundles, such as from a star at nearly-infinite distance - and turn them into images at a finite distance, or vice versa, turning images into collimated beams. In the case of collimated beams, a mirror or lens turns the off-axis angle θ of an incident beam into an off-axis displacement r of the image in the focal plane, and the amount of the displacement is governed by the lens focal length f: $r_{offaxis} = \theta_{offaxis} f$, where θ is in radians.

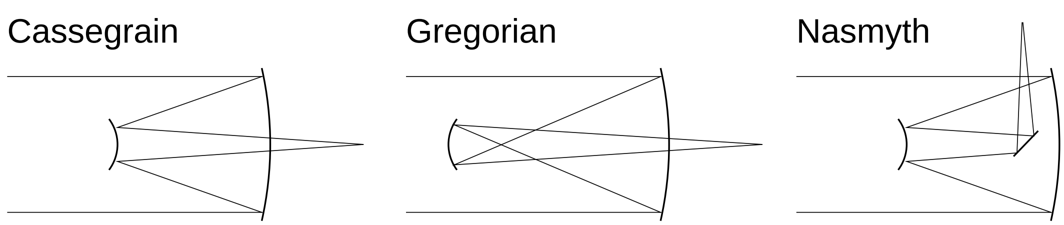

Figure 1: Telescope designs with locations of the secondary focus. The difference between Cassegrain and Gregorian is in the location and curvature of the secondary mirror, and in the field curvature of the focal plane. The Nasmyth focus is a variant of either, using a flat tertiary to change the physical location of the focus. Nasmyth foci are useful in alt-azimuth mounted telescopes.

Large telescopes are usually derived from a Cassegrain or Gregorian design; newer designs are almost all alt-azimuth mounting and frequently use a flat tertiary to provide a Nasmyth focus. Cassegrain telescopes have a convex secondary mirror, while Gregorians have a concave secondary which is located past the primary focus. Cassegrain designs are more common because the overall structure is shorter and the enclosure can be smaller. However, the Gregorian has a focal plane which is concave away from the telescope, toward the instrument, while Cassegrains are the opposite. This sense of curvature can make it easier to design wide field imagers for a Gregorian.[1]

The primary mirror in modern large telescopes is quite fast, with f-number $\sim 1 - 3$, but the secondary mirror slows the system down, with f/5 to f/15 being a common range. In Figure 1, note that the angle of convergence at the secondary focus is narrower than it is at the

[1]The prime examples among 8-meter-class telescopes are the Magellan telescopes, which have an f/11 Gregorian secondary.

prime focus. If one regards the beam as a cone of light, the f-number is simply the ratio of height of the cone to the base.

The plate scale at the secondary focal plane of a telescope and the physical diameter of the field of view depend on the focal length of the telescope f_{tel} (the factor 206265" converts from radians to arcseconds):

$$s_{tel} = 206265''/f_{tel},$$
$$f_{tel} = D_{tel}N_{tel},$$
$$D_{field} = \theta_{field}/s_{tel}.$$

The scale at the secondary focal plane is usually large enough that it is not a good match for modern detectors. For example, at the f/11 focus of the 6.5-m Magellan, the plate scale is 2.9"/mm. A CCD detector with 15 μm pixels would have 0.043"/pixel, which grossly oversamples a reasonable atmospheric seeing ($\sim 0.6''$).

2.2 Simple reimaging systems

Figure 2 illustrates the optical path through a simplified reimaging system, using collimator and camera lenses to reimage the focal plane onto a detector. The collimator and camera are drawn as simple lenses; in a real system, they would have to be more complex to avoid aberrations and curvature of field. However, the physical sizes of collimated beams, the calculations of pixel scales and so on are mostly just dependent on focal lengths and f-numbers, and are similar for simple and complex systems.

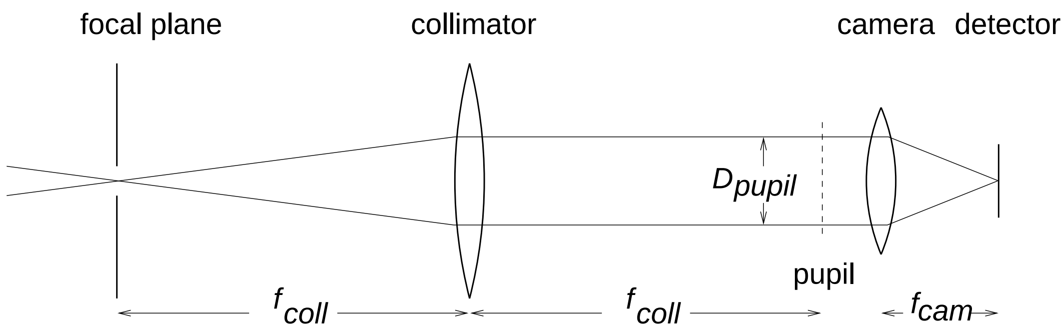

Figure 2: A typical reimaging system mounted behind the focal surface of a telescope, with light entering the focal plane from the left. A beam from a single on-axis object is shown, diverging from the focal plane, recollimated, and reimaged by the camera onto the detector. The collimator and camera have been abstracted as simple lenses and the telescope focal plane is drawn as flat, although in a real system it is not. In some instruments the collimator and (less frequently) the camera use mirrors rather than lenses, but the principles and scaling with focal lengths are similar.

Diverging beams emerge from each point on the focal surface and are re-collimated; the diameter of a collimated beam is the pupil diameter,

$$D_{pupil} = f_{coll}/N_{tel}.$$

The pupil is the aperture of the system, here the primary mirror. The collimator forms an image of the mirror at the pupil location shown by the vertical dashed line. If we were to put a sheet of paper into the beam at this location, we would see a donut of light, the shape of the primary mirror.

2.3 Imager field of view

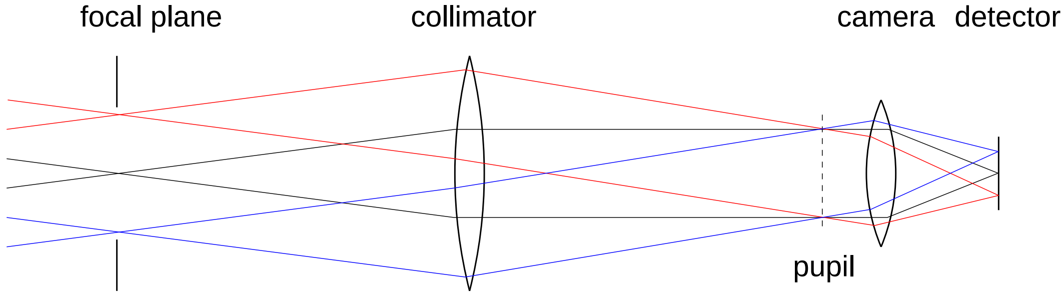

Figure 3: The reimaging system with the paths of light from on-axis (black lines) and off-axis objects (red and blue lines). The beams from the off-axis objects are drawn as parallel to the on-axis beam. These are called "telecentric" beams; real systems are not always telecentric, but the deviations from it are small for the purposes of this discussion. Note that the collimator has to be larger than a single beam to accept off-axis beams, and that all of the beams pass through a "waist" at the pupil.

In order to image a nonzero field of view, the collimator has to accept off-axis beams, and be physically wider than D_{pupil}. Figure 3 shows the path of off-axis beams through the optical system. For the simplified case in which the off-axis beams are parallel to the optical axis ("telecentric") and we have abstracted the collimator as a simple lens, then

$$D_{coll} = D_{pupil} + D_{field}.$$

[Side note: For a complex lens, the D_{coll} isn't necessarily the diameter of all of the collimator glass, but the diameter of the collimator entrance pupil. The entrance pupil of a lens by itself is not the same thing as the pupil of the whole system, indicated by the vertical dotted line. In both cases, pupil means the image of an aperture stop, but for the whole system, the aperture stop is the primary mirror.]

Note that the collimator must have total f-number

$$N_{coll} = f_{coll}/D_{coll},$$

$$N_{coll} = N_{tel}\frac{D_{pupil}}{D_{coll}},$$

$$N_{coll} = N_{tel} \frac{D_{pupil}}{D_{pupil} + D_{field}}.$$

The whole collimator lens is faster than the beam delivered by the telescope, because it has to be wide enough to accept the off-axis beams. Faster lenses or mirrors that accept light from larger off-axis angles are harder to construct. Imaging over a wide field also has to contend with the curvature of the telescope focal plane. In practice, both the size of the available detector and the requirement of good image quality for off-axis points set limits on the field of view of an instrument.

2.4 The pupil

The pupil of the spectrograph is an image of the primary mirror formed by the secondary and the collimator. The pupil diameter D_{pupil} is simply set by the expansion of the beam as it reaches the collimator, as shown in Figure 2, and its location is set by the focal length of the collimator.

$$D_{pupil} = f_{coll}/N_{tel} = \frac{D_{tel} f_{coll}}{f_{tel}}$$

D_{pupil} gives the minimum size of a dispersing element in the collimated beam. This is a critical number since it sets the minimum size of a grating, grism, or other optical device that the instrument requires.

Off-axis beams, which are displaced by $D_{field}/2$ in the focal plane, pass through the pupil at an angle

$$\theta_{offaxis} = \frac{D_{field}}{2 f_{coll}}.$$

All the light from on- and off-axis beams passes through a "waist" at the pupil. If we introduced a screen into the light path at the pupil, we would see an in-focus donut-shaped image of the primary mirror. If the screen were placed ahead of or behind the pupil, the image of the primary would be not clearly focused.

The pupil size interacts with the field of view indirectly. Once the collimator focal length f_{coll} is chosen, increasing the field of view does not increase the pupil size, since all the off-axis beams pass through the "waist" of the pupil. However, if we choose a long f_{coll}, then off-axis light enters the collimator at a less extreme angle, so the collimator lens is slower and easier to design. But long f_{coll} requires a larger pupil. Equivalently, from the equations from N_{coll} above, we see that increasing D_{pupil} relative to D_{field} makes the collimator slower. So although field of view does not depend directly on pupil size, in real optical designs it is difficult to image a large field through a small pupil.

2.5 Reimaging and the scale at the detector

The camera reimages the collimated beams onto the detector/CCD. The camera has to accept the collimated beam so it has

$$D_{cam} = D_{pupil}.$$

In practice, D_{cam} should be slightly greater to avoid vignetting off-axis beams. (This diameter is really the size of the camera's entrance pupil, not the physical size of a lens.) Thus the f-number of the camera is

$$N_{cam} = f_{cam}/D_{pupil} = \frac{N_{tel} f_{cam}}{f_{coll}}.$$

Because the camera focal length is usually fairly short to get demagnification of the focal plane onto a small detector, the camera typically has to be fast (small N_{cam}). As drawn in Figure 3, the beam converges with a wider (faster) angle at the detector, compared to the beam delivered by the telescope.

An off-axis beam is reimaged at distance from the detector center of

$$D_{ccd}/2 = \theta_{offaxis} f_{cam}.$$

So this means that

$$D_{ccd} = D_{field} \frac{f_{cam}}{f_{coll}},$$

$$s_{ccd} = s_{tel} \frac{f_{coll}}{f_{cam}},$$

$$s_{ccd} = \frac{206265''}{f_{tel}} \frac{f_{coll}}{f_{cam}}.$$

The focal plane and plate scale have been demagnified by the ratio of collimator to camera focal lengths. If the collimator is 3x longer than the camera, the detector can be 3x smaller than the field size in the telescope focal plane. Large telescopes have big focal planes, while detectors are usually smaller, so reimaging systems with demagnification allow us to get a reasonable field of view, and can make the pixel scale a better match to the typical seeing.

3 Spectroscopy

Now consider putting a grating or grism in the collimated beam to do spectroscopy. The important choice for spectroscopy is the lines/mm of the grating, call this $M_{grating}$, which governs the spectral resolution. Typical numbers for large astronomical gratings range from 100-1200 lines/mm (apart from echelle gratings, which are used at a different rangle of incident angles and in more complex spectrographs).

3.1 Angles of diffraction: the grating equation

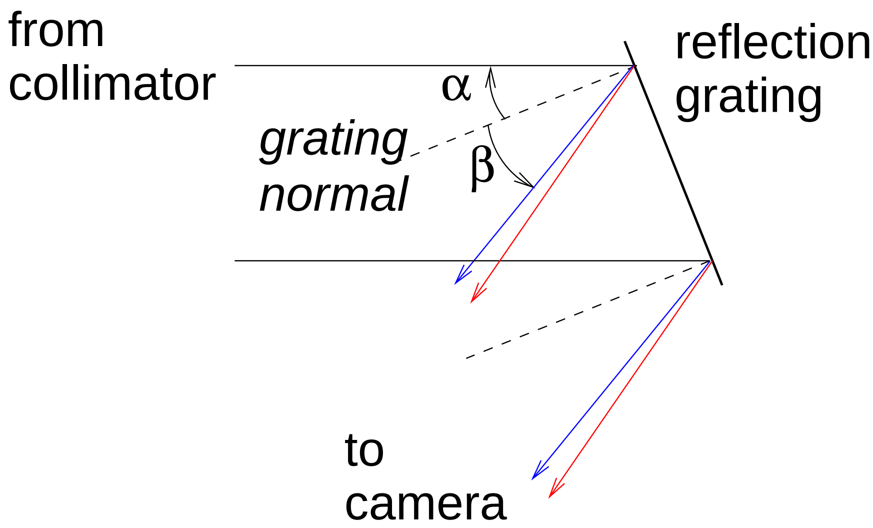

Figure 4: A collimated beam incident from the left on a reflection grating and the outgoing diffracted beams (red and blue). The incident and diffracted angles α and β are governed by the grating equation and depend on wavelength and the lines/mm of the grating.

A transmission grating deviates light of wavelength λ by an angle α,

$$\sin\,\alpha = k_{order}\lambda/l_{grating},$$

where $l_{grating}$ = interline spacing, and k_{order} is an integer ≥ 1. Equivalently,

$$\sin\,\alpha = k_{order}\lambda M_{grating}.$$

Let's work in first order where $k_{order} = 1$. For a reflection grating the grating equation is

$$\sin\,\alpha + \sin\,\beta = k_{order}\lambda M_{grating}$$

where α and β are the angles of incident and diffracted rays with respect to the grating normal, shown in Figure 4. The diffracted beams are shown as red and blue. Light from a single point source produces one incident collimated beam. The diffracted beam of a single point source at a single wavelength (e.g. the blue lines) is still collimated and will be imaged at a single point on the detector, while the red lines will be imaged at a different point. The fact that wavelengths are separated in angle, but each single wavelength stays collimated, is why we want to have the disperser in the collimated beam.

The zeropoint of the diffracted angle β depends on the incident angle and grating normal, but the change in β with λ governs the resolution of the spectrograph.

We're not directly concerned with the zeropoint of β, assuming we have tilted the grating so as to get light into the camera, but with the change in β per wavelength and the resulting wavelength scale per pixel. Consider how the camera translates a deviation in angle of the diffracted beam to a distance on the detector, r_{ccd}. For a small deviation in angle, $d\beta$, the image moves

$$dr_{ccd} = f_{cam}\ d\beta.$$

By differentiating the grating equation, for a fixed input angle α,

$$\cos\beta\ d\beta = M_{grating}\ d\lambda.$$

For typical spectrograph layouts (other than echelles), β is not large and $\cos\beta$ is slightly < 1.

This gives the wavelength/physical scale at the CCD:

$$\frac{d\lambda}{dr_{ccd}} = \frac{\cos\beta}{f_{cam} M_{grating}}.$$

3.2 Spectrograph resolution

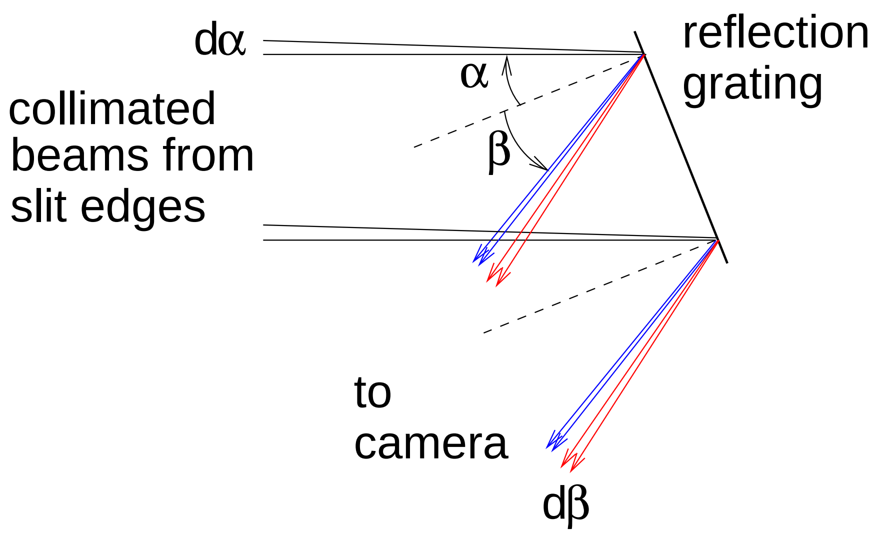

Figure 5: Collimated beams with a small angular separation $d\alpha$, as in the beams from each edge of a slit, are incident from the left on a reflection grating. The outgoing diffracted beams (red and blue) have diffracted angle β that depends on wavelength. At each wavelength, the outgoing beams are spread over a small angle $d\beta$ due to the finite size of the slit.

We know the CCD pixel scale in arcsec on the sky, so this lets us calculate the resolution of the spectrograph for a given slit width in arcsec. Here I will neglect an effect called anamorphic demagnification that modifies the slit width as projected on the detector[2]. Let's assume for simplicity that our spectrograph is configured such that $\alpha \geq \beta$, and $d\alpha \geq d\beta$. (In practice, one would not design a spectrograph so that $\alpha = \beta$ exactly, because the grating would also act as a mirror, reflecting zeroth-order light into the camera.) For some slit width dW in arcsec, if the anamorphic factor $d\alpha/d\beta \sim 1$, dW corresponds to a certain number of detector pixels as calculated earlier.

$$dr_{ccd}(\text{in mm}) = dW/s_{ccd},$$

$$dr_{ccd} = \frac{dW}{206265"} f_{tel} \frac{f_{cam}}{f_{coll}},$$

and we had that

$$d\lambda = \frac{dr_{ccd} \cos \beta}{f_{cam} M_{grating}}.$$

Therefore the delta-wavelength $d\lambda$ for a slit width dW in arcsec is given by

$$d\lambda = \frac{dW}{206265"} \frac{f_{tel}}{f_{coll}} \frac{\cos \beta}{M_{grating}}.$$

Note that the camera focal length has dropped out here and that f_{coll} is in the denominator, meaning longer collimators give higher resolution for a given slit width. This is because the longer collimator translates a given slit width into a smaller spread of angles, hence a smaller spread of wavelength by the grating.

3.3 Spectral resolution is controlled by pupil size

Since $f_{tel} = D_{tel} N_{tel}$, and $f_{coll} = D_{pupil} N_{tel}$, we can rewrite the equation for resolution as a function of slit width as

$$d\lambda = \frac{dW}{206265"} \frac{D_{tel}}{D_{pupil}} \frac{\cos \beta}{M_{grating}}.$$

Many factors have dropped out, leaving only telescope and pupil diameters and the lines/mm of the grating (and cos β, which is not very adjustable). This equation expresses a fundamental relation between telescopes and instruments. Everybody wants a bigger aperture telescope, large D_{tel}, to gather more light. But in order to get equally high resolution spectra, if we increase D_{tel}, we must also increase D_{pupil}. ($M_{grating}$ is limited, since a 1200 lines/mm grating already has interline spacing < 1 micron, close to the wavelengths of the

[2]Anamorphic demagnification comes from the relation of incident to diffracted angles. At fixed wavelength, $\cos \alpha \, d\alpha + \cos \beta \, d\beta = 0$. If the grating tilt is set up such that $\alpha \neq \beta$, then $d\beta \neq d\alpha$. Astronomical spectrographs are typically configured so that $\beta \geq \alpha$ and $d\alpha/d\beta \sim 1 - 1.5$. The diffracted beams from the slit are spread over a smaller angle than the incident beams were, and the slit is demagnified at the detector. This affects the translation of slit size into spectral resolution, giving higher resolution by $\times 1 - 1.5$ than we will calculate for the simple case. See Schweizer (1979, PASP, 91, 149) for a detailed explanation.

light we are trying to diffract; we can't make a high-quality large grating that is significantly finer.)

Again, this is because increasing the telescope size means we have to scale up the instrument, otherwise a given slit passes a larger range of angles to the grating, and that lowers the resolution. If we tried to get around this by making a faster telescope (smaller N_{tel}) with a smaller physical scale s_{tel} at the focal plane, the beam emerging from the focal plane and entering the collimator is faster, so it will make a big pupil anyway.

Note that for given D_{tel}, $d\lambda \propto \frac{1}{D_{pupil} M_{grating}}$. $D_{pupil} M_{grating}$ is the total number of lines in the grating, or the total number of interfering elements; this is a common figure of merit for diffracting systems.

4 Scaling with telescope size

The $d\lambda \propto D_{tel}/D_{pupil}$ scaling means that large telescopes require instruments with large pupils - and that means bigger collimators, cameras, gratings, and detectors. Note that this is true even for a single-object spectrograph where we don't need to image a large field of view, but want a reasonably high resolution.

Constructing large collimators and cameras is a challenge and constructing very large diffraction gratings is extremely difficult. The only way to get around needing a large pupil is to have a narrower slit. That means either suffering slit losses when the slit gets smaller than the seeing, or improving the image resolution delivered to the telescope focal plane, such as with adaptive optics. This is one reason that extremely large telescopes will need and use adaptive optics – to keep some of the *instruments* to a buildable size.

Taking an instrument from a small telescope and putting it on a big telescope can be done (if the telescopes have the same f-number - if the f-number is different, it will either underfill or overfill the pupil). However, it is not ideal. The larger telescope has a larger scale at its focal plane, so for e.g. a 1.0" slit we need a physically wider slit. With the wider slit, the resolution is worse because we are allowing a larger range of angles onto the grating, just as it would be if we sat at the small telescope and opened the slit from 1" to 2".

If we just want to do imaging, we can move a reimaging camera from a small to large telescope, but of course its field of view will be proportionately smaller. The product of $D_{tel}^2 * (field\ diameter)^2$ stays constant, so if we need to map an area larger than the field of view, the large telescope won't be faster - unless it has better image quality.

A number of instruments have used a multi-barrel reimager strategy to cover larger areas. The idea is that for a 2x larger diameter telescope, instead of scaling up a instrument design by 2x diameter (8x the volume of the small instrument, 4x the number of detector pixels), one builds 4 of the smaller spectrographs and tiles the focal plane with them. This covers the same field as the one big spectrograph, with the same number of pixels, and theoretically requires only 4x the volume of the small instrument. Although one has to make 4 of each optical element, the elements are all smaller, so it should be cheaper and less challenging to fabricate. But a drawback is that they are each still using a small pupil, so the resolution of each spectrograph will be limited: for a given slit and grating, the resolution will be 2x worse than it was on the small telescope.

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